TRAJECTORY PLANNING FOR ROBOT MANIPULATORS

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Based on MSc Research by Chinthaka Porawagama

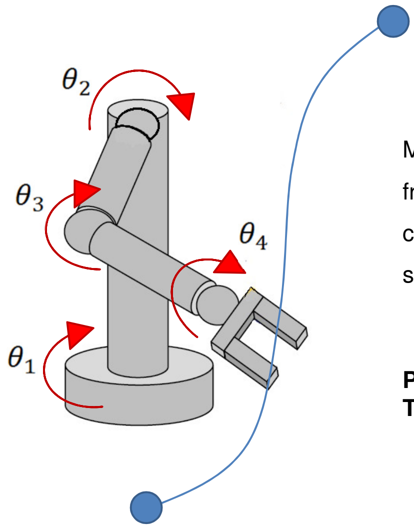
Industrial Robotics Involves in

- Pick-and-place operations
- Assembling operations
- Loading and stacking
- Automated welding, etc.

Proper motion planning is needed in these applications



Trajectory Planning

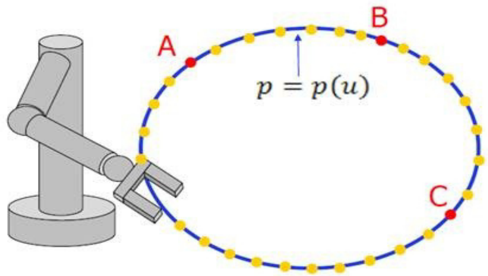


Manipulators with multi degree of freedom for accomplishing various complex manipulation in the work space

Path: only geometric description
Trajectory: timing included

Path Definition

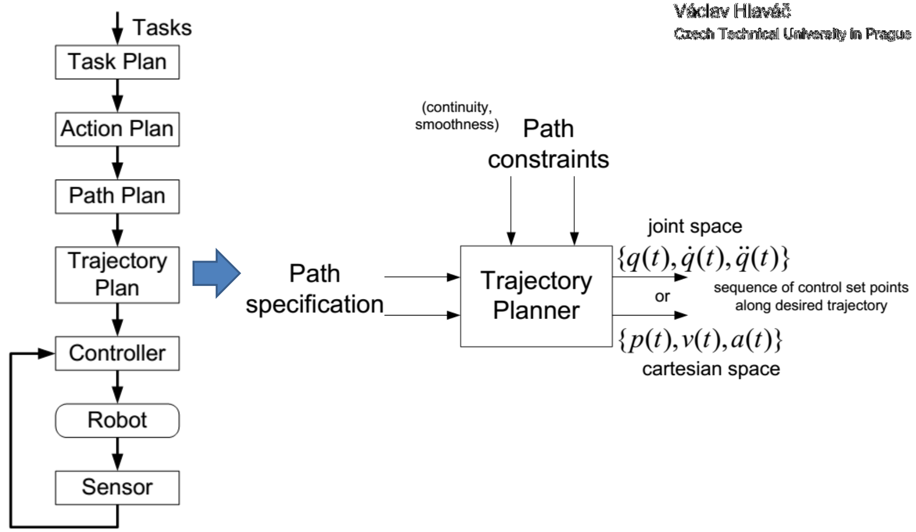
"Expressing the desired positions of a manipulator in the space, as a parametric function of time"



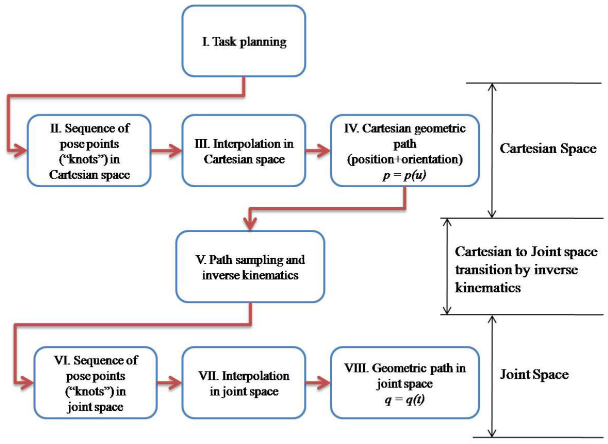
- B. Siciliano et al. Robotics (Modelling, planning and control), Springer, Berlin, 2009, chapter 4: Trajectory planning, pages 161-189.

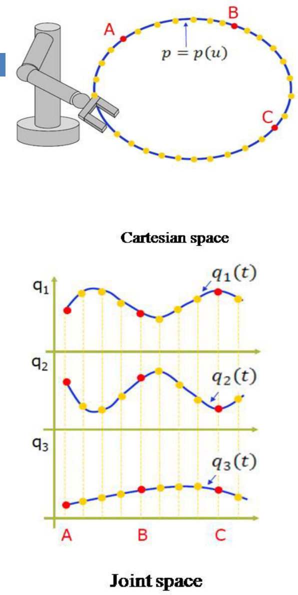
Trajectory Planning

Václav Hlaváč
Czech Technical University in Prague



Task to Trajectory

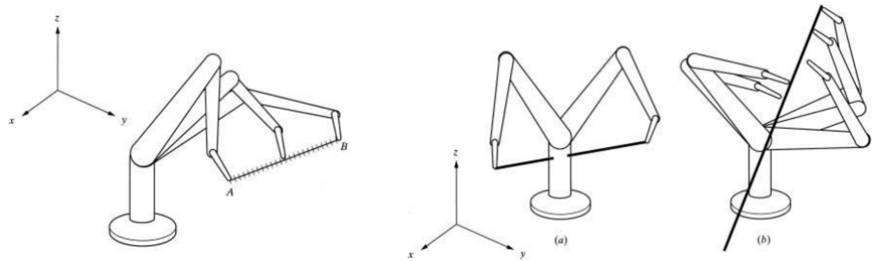




Joint Space Vs Operational Space

- Joint-space description:
  - The description of the motion to be made by the robot by its joint values.
  - The motion between the two points is unpredictable.
- Operational space description:
  - In many cases operational space = Cartesian space.
  - The motion between the two points is known at all times and controllable.
  - It is easy to visualize the trajectory, but it is difficult to ensure that singularity does not occur.

Planning in Operational Space



Sequential motions of a robot to follow a straight line.

Cartesian-space trajectory
(a) The trajectory specified in Cartesian coordinates may force the robot to run into itself, and
(b) the trajectory may requires a sudden change in the joint angles.

Planning in Operational Space

- Calculate path from the initial point to the final point.
- Assign a total time T_{path} to traverse the path.
- Discretize the points in time and space.
- Blend a continuous time function between these points
- Solve inverse kinematics at each step.

- Advantages
  - Collision free path can be obtained.
- Disadvantages
  - Computationally expensive due to inverse kinematics.
  - It is unknown how to set the total time T_{path} .

Planning in Joint Space

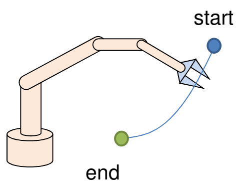
- Calculate inverse kinematics solution from initial point to the final point.
- Assign total time T_{path} using maximal velocities in joints.
- Discretize the individual joint trajectories in time.
- Blend a continuous function between these point.

- Advantages
  - Inverse kinematics is computed only once.
  - Can easily take into account joint angle, velocity constraints.
- Disadvantages
  - Cannot deal with operational space obstacles.

Types of Motion

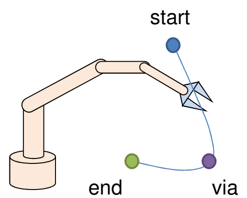
1. Point to point motion:

- End effector moves from a ***start point*** to ***end point*** in work space
- All joints' movements are coordinated for the point-to-point motion
- End effector travels in an arbitrary path



2. Motion with Via Points

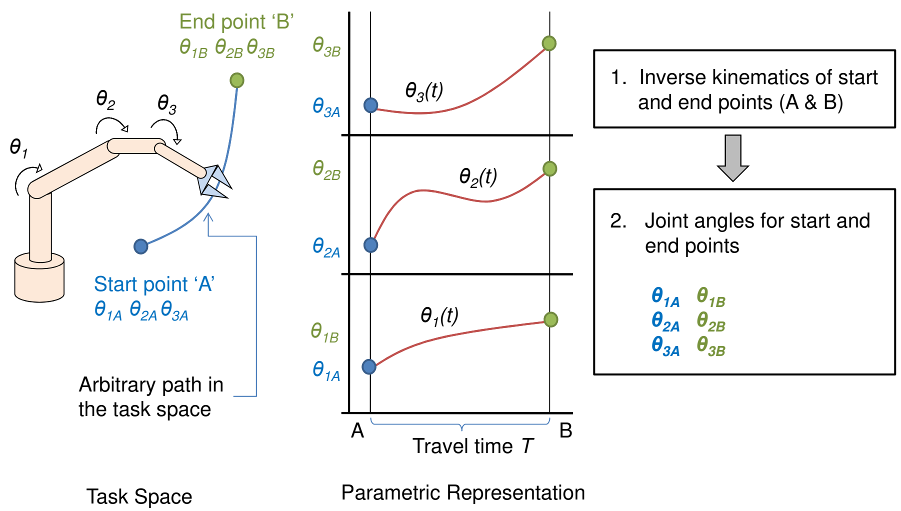
- End effector moves through an intermediate point between start and end
- End effector moves through a via point without stopping



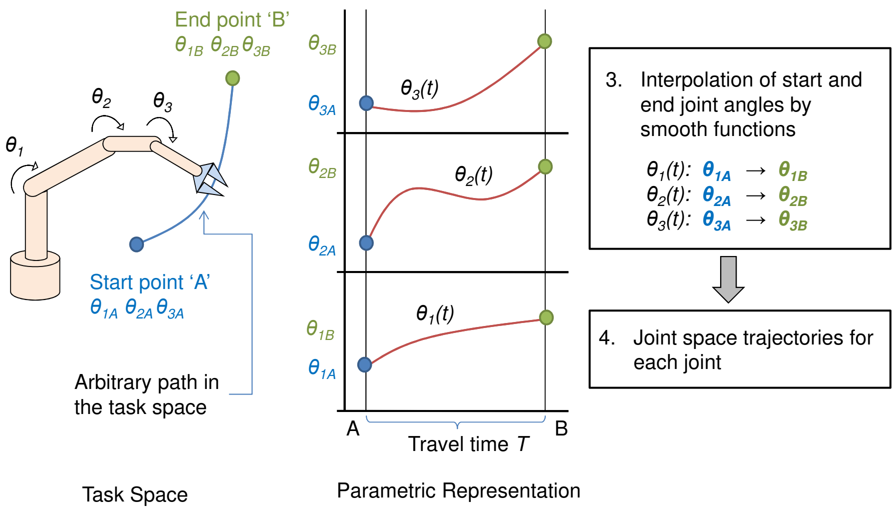
Joint Space Planning

Point to point motion:

"Describing of joints' motions from start to end by smooth functions



Joint Space Planning



Smooth Motion ➡ Quality of Work

- Non smooth trajectories lead to
  - Vibration/jerk
  - Actuator saturation/Path deviation
  - Shorten manipulator lifetime
  - Poor quality in work

Uniform Velocity Trajectory

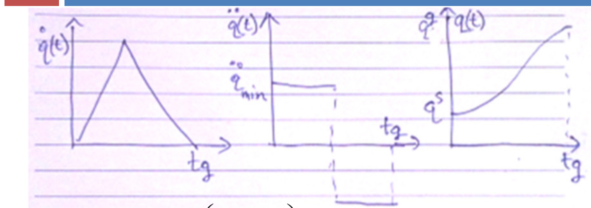
$$q(t) = q^s + \frac{q^g - q^s}{t_g} t$$

$$\dot{q}(t) = \frac{q^g - q^s}{t_g}$$

$$\ddot{q}(t) = \begin{cases} \infty & t = 0, t_g \\ 0 & 0 < t < t_g \end{cases}$$

- Infinite accelerations at endpoints
- Discontinuous velocity when two trajectory segments are connected (at via points)

Triangular Velocity Trajectory



$$q^g - q^s = \frac{1}{2} t_g \left(\ddot{q}_{\min} \frac{t_g}{2} \right)$$

$$q(t) = \begin{cases} q^s + 0.5\ddot{q}_{\min} t^2 & 0 \le t < 0.5 t_g \\ 0.5(q^s + q^g) - 0.5\ddot{q}_{\min} t^2 & 0.5 t_g \le t \le t_g \end{cases}$$

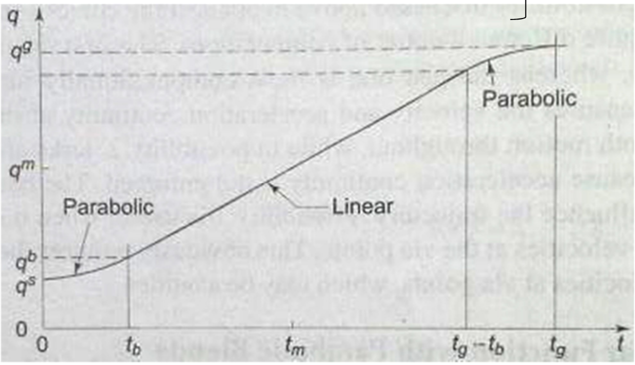
$$\ddot{q}_{\min} = \begin{cases} \frac{4(q^g - q^s)}{t_g^2} & t \le 0.5 t_g \\ -\frac{4(q^g - q^s)}{t_g^2} & 0.5 t_g \le t \le t_g \end{cases}$$

- Acceleration discontinuities exist at endpoints and at the midpoint of the trajectory

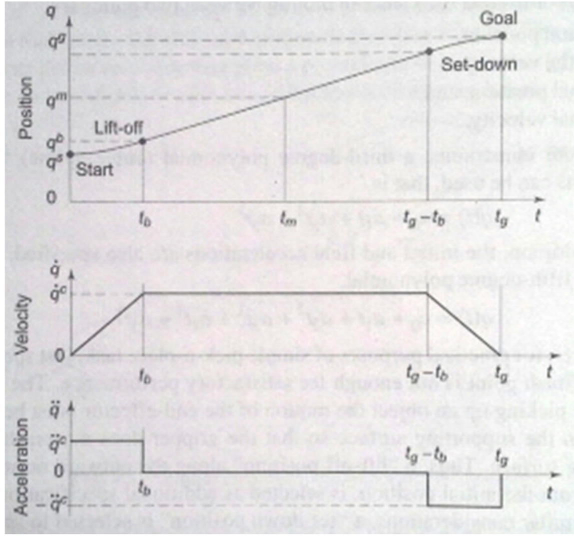
Trapezoidal Velocity Trajectory: Linear Trajectory with Parabolic Blends

- Zero acceleration in middle segments.
- Constant acceleration at end segments.

Acceleration discontinuity at blend points



Trapezoidal Velocity Trajectory: Linear Trajectory with Parabolic Blends



Trapezoidal Velocity Trajectory: Linear Trajectory with Parabolic Blends

- Total angular motion

$$S = 2(parabolic) + Linear$$

$$q^g - q^s = 2 \times \frac{1}{2} t_b \ddot{q}^b t_b + \ddot{q}^b t_b (t_g - 2t_b)$$

$$\ddot{q}^b t_b^2 - \ddot{q}^b t_g t_b + (q^g - q^s) = 0 \rightarrow t_b^2 - t_g t_b + (q^g - q^s)/\ddot{q}^b = 0$$

$$t_b = \frac{t_g}{2} \pm \frac{1}{2}\sqrt{t_g^2 - \frac{4(q^g - q^s)}{\ddot{q}^b}}$$

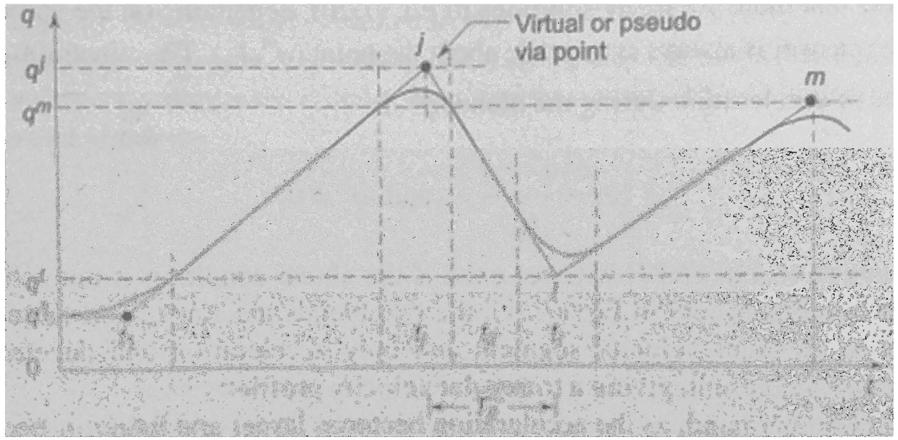
- For a linear part to exist

$$t_g^2 - \frac{4(q^g - q^s)}{\ddot{q}^b} > 0 \Rightarrow \ddot{q}^b > \frac{4(q^g - q^s)}{t_g^2}$$

← Minimum joint acceleration

Trapezoidal Velocity Spline Trajectory

- Multi-stage linear parabolic blend (spline)



Multi-stage linear parabolic blend (spline)

Cubic Polynomial Trajectory: Improving smoothness

Joint Position

$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$ (1)

Joint Velocity

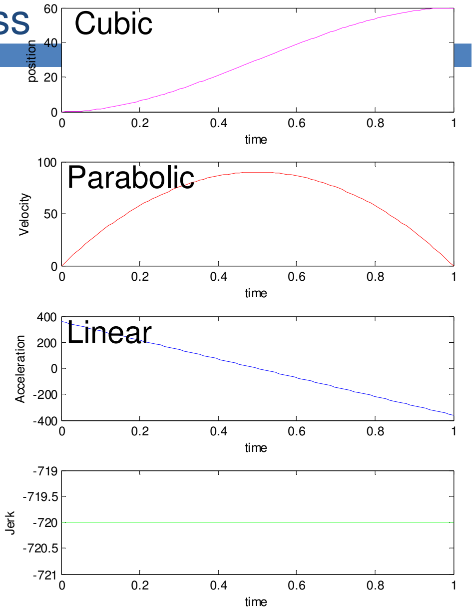
$\dot{q}(t) = a_1 + 2a_2 t + 3a_3 t^2$ (2)

Joint Acceleration

$\ddot{q}(t) = 2a_2 + 6a_3 t$ (3)

Joint Jerk

$\dddot{q}(t) = 6a_3$



Cubic Polynomial Trajectory

- Boundary Conditions for position

$q_s = a_0 \quad (4)$

$q_g = a_0 + a_1 T + a_2 T^2 + a_3 T^3 \quad (5)$

- Boundary conditions for speed

$\dot{q}_s = a_1 \quad (6)$

$\dot{q}_g = a_1 + 2a_2 T + 3a_3 T^2 \quad (7)$

Determine the four parameters

- What about joint accelerations?

$\ddot{q}_s = 2a_2 \quad (8)$

$\ddot{q}_g = 2a_2 + 6a_3 T \quad (9)$

Uncontrollable

Cubic Polynomial Trajectory

- Parameter Calculation

$(4),(6) \Rightarrow (5),$

$q_e = q_s + \dot{q}_s T + a_2 T^2 + a_3 T^3$

$q_e - q_s - \dot{q}_s T = a_2 T^2 + a_3 T^3 \quad (10)$

$(4),(6) \Rightarrow (7)$

$\dot{q}_e = \dot{q}_s + 2a_2 T + 3a_3 T^2$

$\dot{q}_e - \dot{q}_s = 2a_2 T + 3a_3 T^2 \quad (11)$

$$a_2 = \frac{3(q_e - q_s - \dot{q}_s T) - (\dot{q}_e - \dot{q}_s)}{3T^2 - 2T} \qquad a_3 = \frac{(\dot{q}_e - \dot{q}_s) - 2(q_e - q_s - \dot{q}_s T)}{3T^2 - 2T^3}$$

Cubic Polynomial with zero speed at end-points

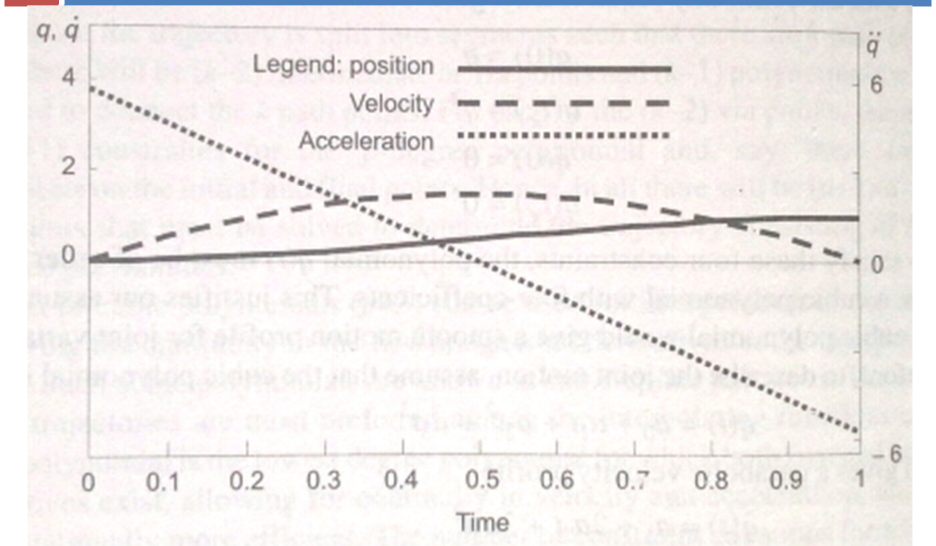
$a_0 = \theta_0$

$a_1 = \dot{\theta}_0 = 0$

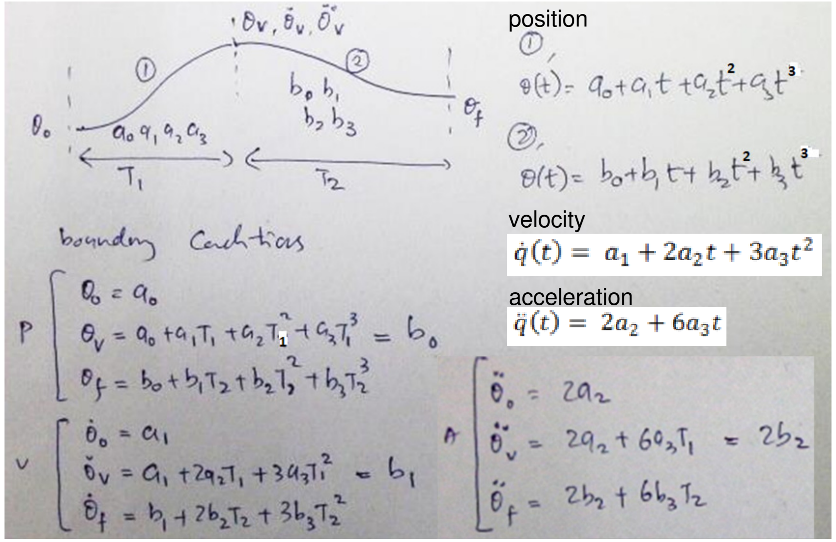
$$a_2 = \frac{3(\theta_f - \theta_0 - \dot{\theta}T) - (\dot{\theta}_f - \dot{\theta}_0)}{3T^2 - 2T} = \frac{3(\theta_f - \theta_0)}{3T^2 - 2T}$$

$$a_3 = \frac{(\dot{\theta}_f - \dot{\theta}_0) - 2(\theta_f - \theta_0 - \dot{\theta}T)}{(3T^2 - 2T^3)} = \frac{-2(\theta_f - \theta_0)}{3T^2 - 2T^3}$$

Cubic Polynomial with zero speed at end-points



Cubic Splines: Piecewise Cubic Polynomial Fitting



PTP with one Via Point

- Plan the trajectory

	start	via point	end
Pos	given	To be continuous	given
Vel	given=0	To be continuous	given=0
Acc	given=0	Not constrained	given=0

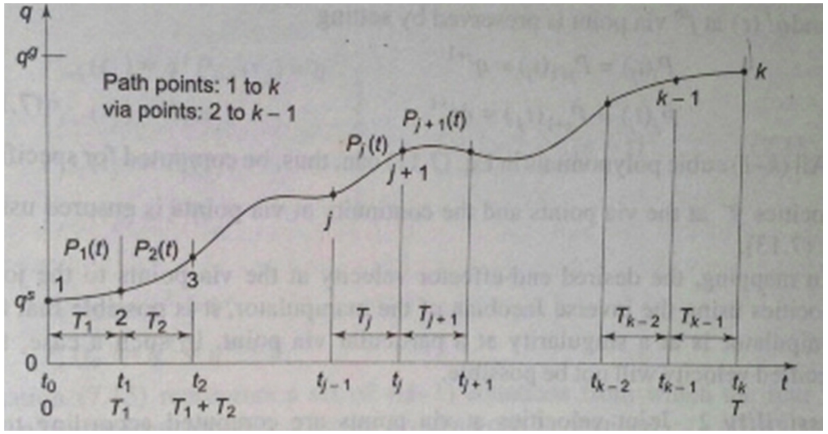
8 boundary conditions (equations)

a0	a1	a2	a3
b0	b1	b2	b3

8 Parameters

Cubic Spline Trajectory

- Stitching cubic polynomials together



5th Order Polynomial: Controls higher order derivatives of the trajectory

$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$

$\dot{q}(t) = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4$

$\ddot{q}(t) = 2a_2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3$

$\dddot{q}(t) = 6a_3 + 24a_4 t + 60a_5 t^2$